



# SEMINAR

## SCHOOL OF MATHEMATICS AND STATISTICS

DATE: 14 DECEMBER 2018

### TITLE

Borel complexity of normal numbers via generic points in subshifts with specification

### VENUE | TIME

Seminar Room I  
03:45 P.M.– 04:45 P.M.

### SPEAKER

Dr. Dominik Kwietniak  
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Poland.

### ABSTRACT

We study the Borel complexity of sets of normal numbers in several numeration systems. Taking a dynamical point of view, we offer a unified treatment for continued fraction expansions and base  $b$  expansions, and their various generalisations: generalised Lüroth series expansions and  $\beta$ -expansions. In fact, we consider subshifts over a countable alphabet generated by all possible expansions of numbers in  $[0, 1)$ . Then normal numbers correspond to generic points of shift-invariant measures. It turns out that for these subshifts the set of generic points for a shift-invariant probability measure is precisely at the third level of the Borel hierarchy (it is a  $\mathbf{f}\pi_3^0$ -complete set, meaning that it is a countable intersection of  $F_\sigma$ -sets, but it is not possible to write it as a countable union of  $G_\delta$ -sets). We also solve Sharkovsky–Sivak problem on Borel complexity of the basin of statistical attraction. The crucial dynamical feature we need is a feeble form of specification. All expansions named above generate subshifts with this property. Hence sets of normal numbers under consideration are  $\pi_3^0$ -complete. The talk is based on a joint work with: Dylan Airey, Steve Jackson, and Bill Mance.