<u>Abstract</u>: Cusp forms are very important holomorphic functions, defined on the complex upper half plane. They carry a lot of arithmetic information, and are intrinsically related to the theory of L-function.

Optimal estimates of cusp forms are very essential in the context of various famous conjectures, like the Arithmetic Quantum Unique Ergodicity Conjecture (AQUE conjecture) for example.

In this talk, we discuss the geometric-analytic methods with we have employed to derive estimates of cusp forms. As an application, we prove a logarithmic version of the AQUE conjecture for compact hyperbolic Riemann surfaces.