SECTION A

- A right answer gets **1 mark** and **0.33 marks** are deducted for a wrong answer.

1. Eight people A,B,...,H, are arranged randomly in a line, the probability that A and B are not next to each other is

   (A) $\frac{3}{4}$.
   (B) $\frac{1}{4}$.
   (C) $\frac{1}{8}$.
   (D) $\frac{3}{8}$.

2. Of all row arrangements of 6 boys and girls, the number of arrangements with at least 3 boys is

   (A) 12.
   (B) 22.
   (C) 32.
   (D) 42.

3. For a population distribution the three quartiles Q_1, Q_2, Q_3 satisfy the relation

$$Q_2 - Q_1 < \frac{1}{3}(Q_3 - Q_2)$$

   Hence the distribution is

   (A) Normal(0,1).
   (B) Students t with 3 degrees of freedom.
   (C) Positively skewed.
   (D) Negatively skewed.

4. The mean of 12 numbers is 21 and their median is 19. Suppose the largest number is increased by 4 and the smallest number is reduced by 7, the mean and median of the modified numbers are, respectively

   (A) 20.75, 19.
   (B) 21, 18.75.
   (C) 20.75, 18.75.
   (D) 21, 19.

5. Let s_1 and s_2 denote the standard deviations of the two sets of data given below respectively,

Set 1	1	4	8	3	7	12
Set 2	1.1	4.4	8.8	3.3	7.7	13.2

,

then

(A) s_2 is 10% more than s_1.

(B) s_2 is 11% more than s_1.

(C) s_2 is 21% more than s_1.

(D) s_2 is equal to s_1.

The next two questions are based on the two data sets given below.

Set 1	10	15	20	25	30
Set 2	6	11	16	21	24

6. Let M_1 & M_2 be the means and s_1 & s_2 be the standard deviations of set 1 and set 2 respectively then

(A) $M_1 > M_2, s_1 > s_2$.

(B) $M_1 > M_2, s_1 < s_2$.

(C) $M_1 < M_2, s_1 < s_2$.

(D) $M_1 > M_2, s_1 = s_2$.

7. If set 1 and set 2 are marks of 5 students in Probability and Analysis respectively and ρ is the correlation coefficient between the marks in the two subjects, then

(A) $\rho = 0$.

(B) $0 < \rho < 1$.

(C) $-1 < \rho < 0$.

(D) $\rho = 1$.

8. A car traveled the first 100 km at a speed of 40 km/hr and the next 100 km at a speed of 60 km/hr. The average speed of the car during the whole journey of 200 km is

(A) 48 km/hr.

(B) 50 km/hr.

(C) $20\sqrt{6}$ km/hr.

(D) 55 km/hr.

9. For a random variable X, the value of b that minimizes $E(|X-b|)$ is

   (A) The modal value of the random variable X.
   (B) The median of the random variable X.
   (C) The mean of the random variable X.
   (D) The first absolute moment ($E(|X|)$) of the random variable X.

10. In an auditorium there are 500 people who were all born in the year 1988. The probability that exactly two of them have the same birthday is

   (A) 1.
   (B) $\frac{365\times364}{366^2}$.
   (C) 0.
   (D) $\left(\frac{365}{366}\right)^2$.

11. A bag contains 100 slips numbered 1,2,. . ., 100. Two slips are drawn without replacement. Let E be the event that the sum of the two numbers on the drawn slips is even and let F denote the event that the sum is an odd number. Then

   (A) $E^C \subset F$.
   (B) $P(E) < P(F)$.
   (C) $P(E) = P(F)$.
   (D) $P(E) > P(F)$.

12. In a locality, there are 100 families with 2 children each. The expected number of families among them where both the children are girls is

   (A) 0.
   (B) 25.
   (C) 50.
   (D) 75.

13. Let X_1, X_2, X_3 be i.i.d. random variables with variance σ^2 and let $Y_1 = X_1 + X_2, Y_2 = X_2 + X_3$. Then the correlation coefficient between Y_1 and Y_2 is

   (A) 0.
   (B) $\frac{1}{4}$.
   (C) $\frac{1}{2}$.
   (D) 1.

14. The p.d.f. of a random variable is given by

$$f(x) = \begin{cases} \theta e^{\theta x}, & x > 0, \theta > 0 \\ 0, & otherwise. \end{cases}$$

Then the value of $f\left(E(X)\right)$ is

(A) 1.

(B) θ.

(C) $\frac{\theta}{e}$.

(D) θe.

15. Let $f(x)$ be the probability density function of a continuous random variable. Then which of the following statements is wrong?

(A) $f(x)$ can not be negative.

(B) $f(x)$ may be larger than 1.

(C) $P\left(X = x\right) = f(x)$.

(D) Exactly one of the statements (a), (b) and (c) is wrong.

16. Let X be a random variable with probability mass function $P\left(X = x\right) = (1-p)^{x-1}p, 0 < p < 1, x = 1, 2, \ldots$. Then $F_X(x)$ is

(A) $1 - (1-p)^x$.

(B) $1 - (1-p)^{x-1}$.

(C) $(1-p)^x$.

(D) $(1-p)^{x-1}$.

17. T_1 and T_2 are independent unbiased estimators of θ and $V(T_2) = V(T_1)$, consider two more estimators for θ- $T_3 = \frac{T_1+T_2}{2}, T_4 = \frac{2T_1+T_2}{3}$. Then

(A) neither T_3 nor T_4 is unbiased for θ.

(B) Both T_3 and T_4 are unbiased estimators for θ and T_3 is more efficient than T_4.

(C) Among T_1, T_2, T_3 and T_4, T_1 is the most efficient.

(D) Among T_1, T_2, T_3 and T_4, T_4 is the most efficient.

18. X is a random variable with mean 1 and standard deviation 3. The $P\left(-4 \leq X \leq 6\right)$ is

(A) $\frac{16}{25}$.

(B) at least $\frac{16}{25}$.

(C) $\frac{9}{25}$.

(D) at most $\frac{9}{25}$.

19. Consider the linear regression model

$$Y_i = \alpha + \beta X_i + \epsilon_i, i = 1, \ldots, n$$

where $E(\epsilon_i) = 0, Var(\epsilon_i) = \sigma^2$ and ϵ_i are uncorrelated. If $\widehat{\alpha}$ and $\widehat{\beta}$ are least squares estimate of α and β respectively, and $\bar{Y}$ is the mean of $Y_i, i = 1, \ldots, n$, an unbiased estimate of σ^2 is

(A) $\frac{1}{n-1} \sum_{i=1}^{n} \left(Y_i - \bar{Y}\right)^2$.

(B) $\frac{1}{n-1} \sum_{i=1}^{n} \left(Y_i - \widehat{\alpha} - \widehat{\beta} X_i\right)^2$.

(C) $\frac{1}{n} \sum_{i=1}^{n} \left(Y_i - \bar{Y}\right)^2$.

(D) $\frac{1}{n-2} \sum_{i=1}^{n} \left(Y_i - \widehat{\alpha} - \widehat{\beta} X_i\right)^2$.

20. The value of a so that the vectors $\mathbf{x}_1^{'} = (2, 3, -5), \mathbf{x}_2^{'} = (5, 2, -7)$ and $\mathbf{x}_3^{'} = (-3, 1, a)$ are linearly dependent is

(A) 0.

(B) 1.

(C) 2.

(D) 3.

21. Suppose $|A|$ is the determinant of a 3×3 nonsingular matrix A. A matrix B is obtained from the matrix A by multiplying the first row of A by -1 and the second row of A by -2. Then the determinant of B is

(A) $|A|$.

(B) $2|A|$.

(C) -2$|A|$.

(D) -$|A|$.

22. For $x \geq 0$, let $\lfloor x \rfloor$ denote the largest integer less than or equal to x. Define a function f on $[0, \infty)$ as follows $f(x) = x - \lfloor x \rfloor$, then

(A) f is continuous everywhere in the interval $[0, \infty)$.

(B) f takes the value 2 for infinitely many values of x.

(C) f is never 0.

(D) f takes the value $\frac{1}{2}$ for infinitely many values of x.

23. The negation of the statement ‘ $a_j > c$ for some $j = 1, 2, \ldots$’ is

(A) ‘$a_j \leq c$ for some $j = 1, 2, \ldots$’.

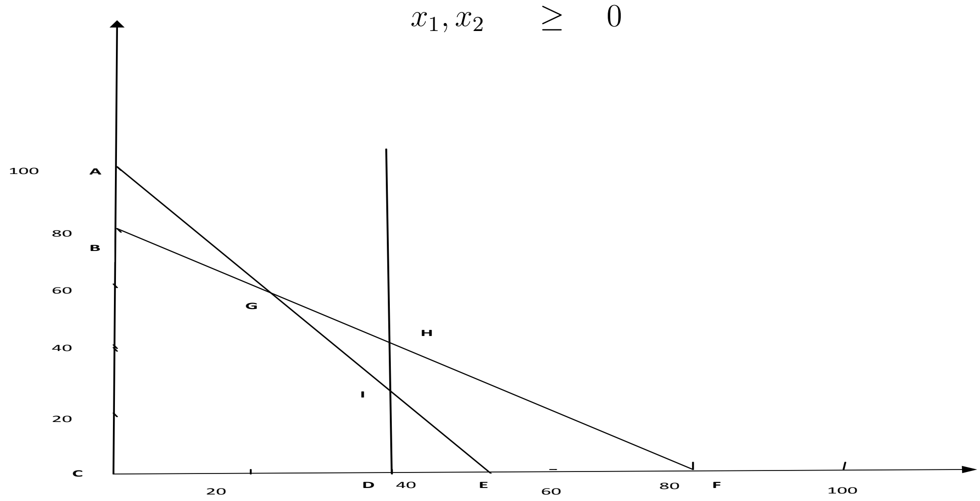
(B) ‘$a_j > c$ for at least one $j, j = 1, 2, \ldots$’.

(C) ‘$a_j \leq c$ for all $j = 1, 2, \ldots$’.

(D) ‘$a_j = c$ for all $j = 1, 2, \ldots$’.

- The next two questions are based on the following Linear Programming Problem and the corresponding graph.

$$\begin{array}{rccc} \max & 3x_1 + 2x_2 & & \\ \text{such that} & 2x_1 + x_2 & \leq & 100 \\ & x_1 + x_2 & \leq & 80 \\ & x_1 & \leq & 40 \\ & x_1, x_2 & \geq & 0 \end{array}$$



24. The feasible region of the Linear Programming Problem included the following points

(A) I,D and E.

(B) E,F, G and I.

(C) G, H and I.

(D) B,D,G, I and C.

25. The optimal solution for the problem is at

(A) Point G and Point H.

(B) Does not exist for the problem.

(C) Point G only.

(D) Point A only.

SECTION B

- A right answer gets **2 marks** and **0.66 marks** are deducted for a wrong answer.

26. In a club 18 members play football, 20 play cricket,19 play hockey, 6 play football and hockey, 7 play hockey and cricket, 5 play cricket and football and 2 play all the 3 games. The number of members in this club is

    (A) 77.

    (B) 65.

    (C) 62.

    (D) 41.

27. Draw two cards from an ordinary pack of cards. The probability that not both are kings is

    (A) $\frac{219}{220}$.

    (B) $\frac{220}{221}$.

    (C) $\frac{222}{223}$.

    (D) $\frac{225}{256}$.

28. If $P(A|B) = 0.75$; $P(A|B^c) = 0.6$, $P(B) = 0.25$, then $P(B^c|A^c)$

    (A) is at most $\frac{1}{4}$.

    (B) is more than $\frac{1}{4}$ but strictly less than $\frac{1}{2}$.

    (C) is more than $\frac{1}{2}$.

    (D) cannot be determined from the given information.

29. Ashok can not find his keys, he has left them at home or in office or has dropped them in the bus on the way to office, with probabilities 0.4, 0.35 and 0.25 respectively. If he has left them at home, the probability of finding them is 1, if he left them at office the probability of finding them is 0.8 and if has dropped them in the bus, the probability of finding them is 0.1. What are the chance of his finding the keys?

    (A) 0.5

    (B) 0.705

    (C) 0.295

    (D) 0.815

30. An urn contains 100 red balls and another urn contains 100 blue balls. The balls are numbered from 1 to 100 in each urn. One ball is drawn from each of the urns, the probability that the number on the red ball is more than the number on the blue ball is

(A) $\frac{1}{200}$.
(B) $\frac{1}{100}$.
(C) $\frac{99}{200}$.
(D) $\frac{101}{200}$.

31. Toss a fair coin twice in a row, Let E_1 denote the event of head showing up in the first toss, E_2 the event of tail showing up in the first toss and E_3 the event of exactly one head in the two tosses, which of the following is correct?

(A) E_1, E_2 and E_3 are pairwise independent but not mutually independent.
(B) E_1, E_2 and E_3 are pairwise independent and mutually independent.
(C) E_1, E_2 and E_3 are mutually independent but not pairwise independent.
(D) E_1, E_2 and E_3 are neither pairwise independent nor mutually independent.

32. A non empty subset is to be drawn from a set of 10 objects. Let E_1 be the event that the subset drawn consists of an even number of objects and E_2 the event that the subset drawn consists of an odd number of objects. If every subset is equally likely to be selected, then

(A) n $P(E_1) < P(E_2)$.
(B) $P(E_1) = P(E_2)$.
(C) $P(E_1) > P(E_2)$.
(D) $P(E_1) + P(E_2) \neq 1$.

33. For two observations, arithmetic and harmonic means are 16 and 9 respectively. The geometric mean of these two observations is

(A) 17.
(B) 15.
(C) 12.
(D) 10.

34. The range of 3 distinct non-negative integers is 3. The variance of these 3 numbers

(A) can not be determined based on the information given.
(B) can be determined only if the mean is known.
(C) can be determined only if at least one of 3 numbers is known.
(D) can be determined.

35. Suppose $X_1 \sim B(n, p_1)$ and $X_2 \sim B(n, p_2)$, $p_2 > p_1 > \frac{1}{2}$, then

(A) $E(X_1) < E(X_2)$ and $Var(X_1) < Var(X_2)$.
(B) $E(X_1) > E(X_2)$ and $Var(X_1) > Var(X_2)$.
(C) $E(X_1) > E(X_2)$ and $Var(X_1) = Var(X_2)$.
(D) $E(X_1) < E(X_2)$ and $Var(X_1) > Var(X_2)$.

36. If $X_1 \sim P(\lambda_1)$, $X_2 \sim P(\lambda_2)$ and $P(X_1 \geq 1) = P(X_2 \geq 2)$, then

(A) $E(X_1) = E(X_2)$.
(B) $E(X_1) < E(X_2)$.
(C) $E(X_1) > E(X_2)$.
(D) $P(X_1 = 0) < P(X_2 = 0)$.

37. Let X be a random variable whose probability mass function is

$$P(X = x) = \frac{1}{3}\left(\frac{2}{3}\right)^n, \quad n = 0, 1, 2, ...$$

Then $P(X = 11 | X > 10)$ is

(A) $\frac{1}{3}$.
(B) $\frac{2}{3}$.
(C) $(\frac{1}{3})^{10}$.
(D) $(\frac{2}{3})^{10}$

38. For a random variable $Z \sim N(0, 1)$, the information available is

$$P(-1.5 < Z < 1.5) = 0.8664, P(Z < 0.29)] = 0.6141$$

Based on the given information the $P(-1.5 < Z < 0.29)$

(A) can not be found.
(B) is equal to 0.5473.
(C) is at least 0.6141.
(D) is at least 0.8664.

39. A computer is programmed to give single digit numbers between 0 and 9 inclusive in such a way that the probability of getting an odd digit $\{1, 3, 5, 7, 9\}$ is half the probability of getting an even digit $\{0, 2, 4, 6, 8\}$. The expected value of X is

(A) 3.3333.
(B) 3.5.
(C) 4.3333.
(D) 4.5.

40. If X is uniformly distributed on the interval $(-2, 2)$, then $P(|X - 0.5| > 1.5)$ is

(A) $\frac{1}{4}$.

(B) $\frac{1}{3}$.

(C) $\frac{1}{2}$.

(D) $\frac{2}{3}$.

41. A candidate can appear in certain competitive examination for a maximum of 4 attempts. The probability of success for a particular candidate in an attempt is 0.3 . Assume that the outcome of an attempt is independent of the outcomes in other attempts. The expected number of attempts for this candidate is

(A) 2.533.

(B) 2.431.

(C) 2.413

(D) 2.353.

42. A box contains 8 red balls and 2 black balls, for every red ball drawn a player will lose Rs.2 and for every blue ball drawn the player will win Rs.8. The player draws 3 balls without replacement, his expected gains or losses are

(A) 0.

(B) loss of Rs 7.

(C) loss Rs.8.

(D) Gain of Rs.14.

43. A discrete random variable X takes values -1,2 and 3 with $P(X = -1) = 0.2$ and $E(X) = 2.2$. Then $P(X = 2)$ and $P(X = 3)$ respectively are

(A) 0 and 0.8.

(B) 0.2 and 0.6.

(C) 0.4 and 0.4.

(D) 0.8 and 0.

44. Let X_1, X_2,...X_n be a random sample from the Poisson population with parameter λ population. Which one of the following statistics is an unbiased estimator for λ^2?

(A) $(\overline{X})^2$.

(B) $\overline{X}(\overline{X} - 1)$.

(C) $\frac{1}{n}\sum_1^n X_i^2$.

(D) $\frac{1}{n}\sum_1^n X_i(X_i - 1)$

45. Let X be discrete random variable on $\{-1, 1, 2\}$ with probability mass function $P_\theta(X = x)$, $\theta \in \{\theta_0, \theta_1\}$ as given below

$\theta \backslash X$	-1	1	2
θ_0	0.3	0.4	0.3
θ_1	0.5	0.2	0.3

If $X_1 = -1$ and $X_2 = 2$ are random observations on X, then the maximum likelihood estimate of θ

(A) 0.5.

(B) θ_0.

(C) θ_1

(D) cannot be calculated.

46. Based on the random sample of size 9 from $N(\mu, \sigma^2 = 1)$ population $H_0 : \mu = \mu_0$ is rejected in favour of $H_1 : \mu_1 > \mu_0$, if sample mean > 12 at 0.05 level of significance(l.s). It was later discovered that the population variance is 4 and not 1. For the same observations and critical region, using the fact that the random sample is now from $N(\mu, \sigma^2 = 4)$

(A) the l.s is less than 0.05.

(B) the l.s is more than 0.05.

(C) the power of the test will increase.

(D) the power of the test will decrease.

47. If frequencies of all classes which are of equal length are equal to 7, the value of the Chi-square statistic for testing for uniformity is

(A) 0.

(B) $\frac{1}{n}$, where n is the number of observations.

(C) $\frac{1}{m}$, where m is the number of classes.

(D) 1.

48. The rank of the matrix

$$\begin{pmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{pmatrix}$$

(A) is 1.

(B) is 2.

(C) is 3.

(D) is 4.

49. Let $\{a_n\}$ be a sequence of real numbers. If $a_n = \frac{(-1)^n}{n}$, $n = 1, 2, ...,$ then $\sum_{n=1}^{\infty} a_n$

(A) does not exist.

(B) diverges to $+\infty$.

(C) diverges to $-\infty$.

(D) converges.

50. The value of $\sum\sum_{1 \leq i < j \leq 10} ij$ is

(A) 1300.

(B) 1320.

(C) 1340.

(D) 1360.